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7/9/25

(a) $y=x$, $y=x^3$

$x=x^3$

$x^3-x=0$

$x(x^2-1)=0$

$x(x+1)(x-1)=0$

$\Rightarrow x=-1, 0, 1$

$f(x)=x-x^3$

$$\text{Average } x=\frac{\int_0^1 x(x-x^3)\,dx}{\int_0^1 (x-x^3)\,dx}$$

$$=\frac{x^3/3-x^5/5\Big|_0^1}{x^2/2-x^4/4\Big|_0}$$

$$=\frac{\frac{5-3}{15}}{1/4}$$

$$=\frac{8}{15}$$

(b) $$P\left(x>\frac{1}{2}\right)=\frac{\int_{\frac{1}{2}}^1 x-x^3\,dx}{\int_0^1 x-x^3\,dx}$$

$$=\frac{\frac{x^2}{2}-\frac{x^4}{4}\Big|_{1/2}^1}{\frac{x^2}{2}-\frac{x^4}{4}\Big|_0^1}$$

$$=\frac{\left(\frac{1}{2}-\frac{1}{4}\right)-\left(\frac{1}{8}-\frac{1}{64}\right)}{1/4}$$

$$=\frac{1/4-7/64}{1/4}$$

$$=\frac{9/64}{1/4}=\frac{9}{16}$$